Confining interquark potentials from nonabelian gauge theories coupled to dilaton

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Abstract

Following a recently proposed confinement generating mechanism, we provide a new string inspired model with a massive dilaton and a new dilaton coupling function [5]. By solving analytically the equations of motion, a new class of confining interquark potentials is derived which includes several popular potential forms given in the literature.

keywords: Dilaton; confinement; quark potential.

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1 Overview

Recently, the extension of gauge field theories by inclusion of dilatonic degrees of freedom has evoked considerable interest. Particularly, dilatonic Maxwell and Yang-Mills theories which, under some assumptions, possess stable and finite energy solutions [1]. Indeed, in theories with dilaton fields, the topological structure of the vacuum is drastically changed compared to the non dilatonic ones. It is therefore of great interest to investigate the vacuum solutions induced by r-dependent dilaton field, through a string inspired effective theory which may reproduce the main feature of strong interactions: quark confinement. Recall that the dilaton is an hypothetical scalar particle appearing in the spectrum of string theory and Kaluza-Klein type theories [2]. Along with its pseudo scalar companion, the axion, they are the basis of the discovery F-theory compactification [3] and of the derivation of type IIB self duality [4]. The main features of a dilaton field is its coupling to the gauge fields through the Maxwell and Yang-Mills kinetic term. In particular, in string theory, the dilaton field determines the strength of the gauge coupling at tree level of the effective action. In this context, Dick [6] observed that a superstring inspired coupling of a massive dilaton to the 4d $SU(N_c)$ gauge fields provides a phenomenologically interesting interquark potential $V(r)$ with both the Coulomb and confining phases. The derivation performed in [6] is phenomenologically attractive since it provides a new confinement generating mechanism. In this context, a general formula of a quark-antiquark potential, which is directly related to the dilaton-gluon coupling function, has been obtained in [7]. The importance of this formula is manifest since it generalizes the Coulomb and Dick potentials, and it may be confronted to known descriptions of the confinement, particularly, those describing the complex structure of the vacuum in terms of quarks and gluons condensates.

In this work, we shall propose a new effective coupling of a massive dilaton to chromo-electric and chromomagnetic fields subject to the requirement that the Coulomb problem still admits an analytic solution. Our main interest concerns the derivation of a new family of confining interquark potentials. As a by product, it is shown that several popular phenomenological potentials may emerge from these low energy effective theories.

2 The model

Let us consider an effective field theory defined by the general Lagrangian density:

$$\mathcal{L}(\phi, A) = -\frac{1}{4F(\phi)} G^a_{\mu\nu} G^{\mu\nu}_a + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) + J^\mu_a A^a_\mu \tag{1}$$

where $V(\phi)$ denotes the non perturbative scalar potential of ϕ and $G^{\mu\nu}$ is the field strength in the language of 4d gauge theory.

$F(\phi)$ is the coupling function depending on the dilaton field. Several forms of $F(\phi)$ appeared in different theoretical frameworks: $F(\phi) = e^{-k\frac{\phi}{f}}$ as in string theory and Kaluza-Klein theories [2]; $F(\phi) = \frac{\phi}{f}$ in the Cornwall-Soni model parameterizing the glueball-gluon coupling [8, 9]. As to Dick model, $F(\phi)$ is given by $F(\phi) = k + \frac{f^2}{\phi^2}$. The constant f is a characteristic scale of the strength of the dilaton/glueball-gluon. By using the formal analogy between the Dick problem and the Eguchi-Hansen one [10], we noted in [7] that f is similar to the $4dN = 2$ Fayet-Illioupoulos coupling in the Eguchi-Hansen model. It may be interpreted as the breaking scale of the $U(1)$ symmetry rotating the dilaton field.

To analyze the problem of the Coulomb gauge theory augmented with dilatonic degrees of freedom in (1), we proceed as follows: first, we consider a point like static Coulomb source which is defined in the rest frame by the current:

$$J^\mu_a = g\delta(r)C_a\nu^\mu_0 = \rho_a\eta^\mu_0 \tag{2}$$

The equations of motion, inherited from the model (1) and emerging from the static configuration (2) are given by:

$$\left[D_\mu, F^{-1}(\phi)G^{\mu\nu}\right] = J^\nu \tag{3}$$

and

$$\partial_\mu \partial^\mu \phi = -\frac{\partial V(\phi)}{\partial \phi} - \frac{1}{4}\frac{\partial F^{-1}(\phi)}{\partial \phi} G_a^{\mu\nu} G_a^{\nu\mu} \tag{4}$$

By setting $G_a^{0i} = E^i \chi_a = -\nabla^i \Phi_a$, we derive, after some straightforward algebra, the important formula [7, 11],

$$\Phi_a(r) = \frac{-gC_a}{4\pi} \int dr \frac{F(\phi(r))}{r^2} \tag{5}$$

which shows that confinement appears if the following condition is satisfied:

$$\lim_{r\to\infty} rF^{-1}(\phi(r)) = finite \tag{6}$$

Thereby the interquark potential reads as,

$$U(r) \;\; = \;\; 2\widetilde{\alpha}_s \int \frac{F(\phi(r))}{r^2} dr \tag{7}$$

with $\widetilde{\alpha} = \frac{g^2}{32\pi^2}\left(\frac{N_c-1}{2N_c}\right)$

At this stage, note that the effective charge is defined by,

$$Q^a_{eff}(r) = \left(g\frac{C_a}{4\pi}\right) F(\phi(r))$$

thus the chromo-electric field takes the usual standard form:

$$E_a = \frac{Q^a_{eff}(r)}{r^2}$$

Therefore, it is the running of the effective charge that makes the potential stronger than the Coulomb potential. Indeed if the effective charge did not run, we recover the Coulomb spectrum.

To solve the equations of motion (3) and (4), we need to fix two of the four unknown quantities $\phi(r)$, $F(\phi)$, $V(\phi)$ and $\Phi_a(r)$ in our model. We set $V(\phi)$ to $V(\phi) = \frac{1}{2}m^2\phi$ and introduce a new coupling function:

$$F(\phi) = \left(1 - \beta\frac{\phi^2}{f^2}\right)^{-n}$$

By replacing $F(\phi)$ in equation (4), it becomes very difficult to solve analytically. However since we are usually interested by the large distance behavior of the dilaton field and its impact on the Coulomb problem, an analytical solution in the asymptotic regime is very satisfactory. Indeed, it is easily shown that the following function:

$$\phi = \left[\frac{f^2}{\beta} - \left(\frac{\beta}{f^2}\right)^{\frac{-n}{n+1}} \left(\frac{2n\alpha_s}{m^2}\right)^{\frac{1}{n+1}} \left(\frac{1}{r}\right)^{\frac{4}{n+1}}\right]^{\frac{1}{2}} \tag{8}$$

solves (4) at large r. Therefore, thanks to the master formula (5), we derive the potentials,

$$\Phi_a(r) = -\frac{gC_a}{4\pi}\left(\frac{2n\beta\alpha_s}{m^2 f^2}\right)^{\frac{-4n}{n+1}} \frac{n+1}{3n-1} r^{\left(\frac{3n-1}{n+1}\right)} \tag{9}$$

By imposing the condition (6), we obtain a family of confining interquark potentials if $n \geq \frac{1}{3}$. If moreover, we invoke the criterion of Seiler [12], then the values of n are constrained to the range $n \leq 1$. Therefore the confinement in our model (1) appears for the coupling function $\frac{1}{F(\Phi)}$ with $n \in \left[\frac{1}{3}, 1\right]$. Such class of confining potentials is very attractive. Indeed, by selecting specific values of n, we may reproduce several popular interquark potentials: Indeed if $n = 1$, we recover the confining linear term of Cornell potential [13]. Martin's potential $(V(r) \sim r^{0.1})$[14] corresponds to $n = \frac{11}{29}$, while Song-Lin

interquark potential [15] and Motyka-Zalewski potential [16], with a long range behavior scaling as $\sqrt{r}$, are obtained by setting n to $\frac{3}{5}$. Turin potential [17] is also recovered for $n = \frac{5}{9}$. We see then, that these phenomenological potentials, which gained credibility only through their confrontation to the hadron spectrum, are now supplied with a theoretical basis since they can be derived from a low energy effective theory.

3 Conclusion

we have derived a family of electric solutions corresponding to a string inspired effective gauge theory with a r-dependent massive dilaton and a new coupling function $F(\phi) = \left(1 - \beta\frac{\phi^2}{f^2}\right)^{-n}$. By constraining the values of n via the Seiler criterion and the condition of Eq.(6) we have shown the existence of a class of confining interquark potentials. Also, several popular quark potentials, which are successful in describing meson and baryon spectra, may emerge from such low energy effective theory.

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References

[1] M. Cvetic, A.A. Tseytlin, Nucl. Phys. **B 416**, 137 (1983).

[2] M. Green, J. Schwartz, E. Witten, Superstring Theory, (Cambridge University Press, Cambridge 1987)

[3] C. Vafa, Nucl.Phys. **B 469**, 403 (1996).

[4] A. Sen, Unification of string dualities, Nucl. Phys. Proc. Suppl. **58**, 5 (1997).

[5] M. Chabab, L. Sanhaji, **hep-th/0311096**.

[6] R. Dick, Eur. Phys. J. **C 6**, 701 (1999); Phys. Lett. **B 397**, 193 (1999).

[7] M. Chabab, R. Markazi, E. H. Saidi, Eur. Phys. J. **C 13**, 543 (2000).

[8] J. M. Cornwall, A. Soni, Phys. Rev. **D 29**, 1424 (1984).

[9] R. Dick, L. P. Fulcher, Eur. Phys. J. **C 9**, 271 (1999).

[10] A. Galperin, E. Ivanov, V. Ogievetsky, P.T. Towsend, Class. Quantum Gravity 1, (1985) 469.

[11] M. Chabab et al., Class. Quantum Gravity **18**, 5085 (2001).

[12] E. Seiler, Phys. Rev. **D 18**, 482 (1978).

[13] E. Eichten et al., Phys. Rev. Lett. **34**, 369 (1975).

[14] A. Martin, Phys. Lett. **B 100**, 511 (1981).

[15] X. Song, H. Lin, Z. Phys. **C 34**, 223 (1987).

[16] L. Motyka, K. Zalewski, Z. Phys. **C 69**, 342 (1996).

[17] D. B. Lichtenberg, E. Predazzi, R. Roncaglia, M. Rosso, J.G. Wills, Z. Phys. **C 41**, 615 (1989).